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PULSED DISCHARGE OF A TWO-PHASE MEDIUM FROM A BOUNDED DUCT CAPABLE OF LONGITUDINAL MOTION

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The regularities of the initial stage of discharge of a two-phase disperse medium from a bounded duct capable of longitudinal motion are established. Numerical results obtained within the framework of a medium with nonequilibrium for velocities and temperatures are compared with analytical solutions of the equilibrium mechanics of disperse systems for the case where the characteristic time of the process is of the order of the characteristic times of velocity and thermal relaxation or smaller.

Developing new technologies of fire extinguishing based on the processes of pulsed supply of a fire-extinguishing powder under the action of a pressurized gas requires a comprehensive investigation of the mechanism of such phenomena and predictions of the performances of corresponding devices.

A method of supply uses unsteady discharge from a duct of a two-phase medium [1–9] (a mixture of a pressurized gas and close-packed particles that fill the duct uniformly at the initial time). Lyubarskii and Ivanov [1] report results of studies of one-dimensional, two-phase flow in an immovable duct using an equilibrium (one-velocity approximation) flow model. Lyubarskii et al. [2] studied the effect of duct recoil on the parameters of one-dimensional, equilibrium, unsteady flow of a two-phase medium from the duct. The dynamics of discharge of a gas-disperse mixture into a gas was studied numerically by Kazakov et al. [3] and Kutushev and Rudakov [4] for disperse-phase volume concentrations $\alpha_2 \leqslant 0.16$ and $\alpha_2 \simeq 0.7$, respectively, within the framework of a one-dimensional model of a nonequilibrium, collision-free, two-phase medium as applied to the experimental conditions of [5, 6]. Vorozhtsov et al. [7] and Fedorov [8] analyzed the equations describing the process of sudden ejection of coal and gas taking account of desorption. Ivanov et al. [9] studied the unsteady discharge of a two-phase medium from an immovable cylindrical duct of finite dimensions into the atmosphere in a two-dimensional axisymmetric formulation within the framework of the mechanics of heterogeneous media; they obtained an exact solution of the corresponding model problem (one-dimensional, one-velocity approximation) for a two-phase medium with arbitrary concentration of the disperse phase.

In the present study, we pose two main problems: to establish the regularities of the initial stage of discharge of a two-phase disperse medium from a bounded duct capable of longitudinal motion and to examine the validity of the one-velocity flow model for the problem considered.

Formulation of the Problem. Within the framework of conventional assumptions, the equations of two-dimensional plane motion of a gas-disperse medium taking into account inertial effects in flow around particles [10] can be written as

$$\frac{\partial \rho_i}{\partial t} + \frac{\partial \rho_i v_i^k}{\partial x^k} = 0,$$

$$\frac{\partial \rho_1 \boldsymbol{v}_1}{\partial t} + \frac{\partial \rho_1 v_1^k \boldsymbol{v}_1}{\partial x^k} + \beta_1 \frac{\partial p}{\partial x^k} = -\beta_3 \boldsymbol{F}_\mu + \beta_3 \rho_1 \boldsymbol{g} + (1-\beta_2)(\rho_1+\rho_2)\boldsymbol{g},$$

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$$\frac{\partial \rho_2 \boldsymbol{v}_2}{\partial t} + \frac{\partial \rho_2 v_2^k \boldsymbol{v}_2}{\partial x^k} + (1-\beta_1)\frac{\partial p}{\partial x^k} = \beta_3 \boldsymbol{F}_\mu - \beta_3 \rho_1 \boldsymbol{g} + \beta_2(\rho_1+\rho_2)\boldsymbol{g},$$

$$\frac{\partial \rho_2 u_2}{\partial t} + \frac{\partial \rho_2 u_2 v_2^k}{\partial x^k} = Q, \qquad (1)$$

$$\frac{\partial}{\partial t}(\rho_1 E_1 + \rho_2 E_2) + \frac{\partial}{\partial x^k}[\rho_1 E_1 v_1^k + \rho_2 E_2 v_2^k + p(\alpha_1 v_1^k + \alpha_2 v_2^k)] = \rho_1 \boldsymbol{g}\cdot \boldsymbol{v}_1 + \rho_2 \boldsymbol{g}\cdot \boldsymbol{v}_2,$$

$$\rho_i = \rho_i^0 \alpha_i, \qquad E_i = u_i + (1/2)v_i^2 \qquad (i = 1,\, 2),$$

$$\beta_1 = \frac{\alpha_1(2+\chi_m \rho_1^0/\rho_2^0)}{2+\chi_m(\alpha_2+\alpha_1\rho_1^0/\rho_2^0)}, \quad \beta_2 = \frac{2+\chi_m\alpha_2}{2+\chi_m(\alpha_2+\alpha_1\rho_1^0/\rho_2^0)}, \quad \beta_3 = \frac{2}{2+\chi_m(\alpha_2+\alpha_1\rho_1^0/\rho_2^0)}.$$

Here and below, the subscripts 1 and 2 correspond to the parameters of the carrier and disperse phases, the superscript 0 corresponds to the true values of density, and the superscript $k = 1$ and 2 corresponds to the projections of the vectors onto the directions of Cartesian coordinates. The volumetric fraction, the reduced density, the velocity vector, the total and internal energies of a mass unit of the ith phase, the gas pressure, and the free-fall acceleration vector are denoted by α_i, ρ_i, $\boldsymbol{v}_i$, E_i, u_i, p, and $\boldsymbol{g}$, respectively, $\boldsymbol{F}_\mu$ and Q are the viscous component of the interphase interaction force and the rate of heat exchange between the gas and the particles in a unit volume of the mixture, respectively, χ_m is a coefficient that takes into account the effect of nonsingularity and nonsphericity of the particles on the attached-mass force (for spherical particles, $\chi_m = 1$), and t is time.

The system of quasilinear equations (1) is supplemented by the equations of state for an ideal, calorically perfect gas and incompressible solid particles and by the laws of interphase interaction [10]:

$$p = (\gamma_1 - 1)\rho_1^0 u_1, \quad u_1 = c_v T_1, \quad u_2 = c_2 T_2, \quad \gamma_1, c_v, c_2, \rho_2^0 \equiv \text{const},$$

$$\boldsymbol{F}_\mu = (3/8)(\alpha_2/r)C_\mu \rho_1 \boldsymbol{w}_{12}|\boldsymbol{w}_{12}|, \quad \boldsymbol{w}_{12} = \boldsymbol{v}_1 - \boldsymbol{v}_2, \quad Q = (3/2)(\alpha_2/r^2)\lambda_1 \mathrm{Nu}_1(T_1 - T_2),$$

$$C_\mu = C_\mu(\mathrm{Re}_{12}, \alpha_2), \quad \mathrm{Nu}_1 = \mathrm{Nu}_1(\mathrm{Re}_{12}, \mathrm{Pr}_1), \quad \mathrm{Re}_{12} = 2r\rho_1^0|\boldsymbol{w}_{12}|/\mu_1, \quad \mathrm{Pr}_1 = c_v\gamma_1\mu_1/\lambda_1.$$

Here T_1 and T_2 are the temperatures of the carrier phase and the particles, γ_1, c_v, and c_2 are the adiabatic exponent, the specific heat of the gas with constant volume, and the specific heat of the particles, Re_{12} and Pr_1 are the Reynolds and Prandtl numbers, Nu_1 is the Nusselt number, which is determined empirically as a function of the former two numbers [11], C_μ is the interphase-friction coefficient determined empirically [12], μ_1 and λ_1 are the dynamic viscosity and thermal conductivity of the gas, respectively, and r is the particle radius.

At the initial time, the duct contains an immovable mixture of the pressurized gas and the dispersed particles, and outside the duct there is an unpertubed atmosphere with parameters denoted by subscripts 0 and a, respectively:

$$p = p_0, \quad T_1 = T_2 = T_0, \quad \alpha_1 = \alpha_{10}, \quad \boldsymbol{v}_1 = \boldsymbol{v}_2 = 0,$$

$$p = p_a, \quad T_1 = T_2 = T_a, \quad \alpha_1 = 1, \quad \boldsymbol{v}_1 = \boldsymbol{v}_2 = 0.$$

Rupture of the diaphragm separating the gas-disperse mixture from the ambient atmosphere causes discharge of the two-phase medium and motion of the duct in the opposite direction under a certain law, which are to be calculated. The problem is solved for the following initial data: $p_0 = 5$ MPa, $p_a = 0.1$ MPa, $T_{i0} = T_{ia} = 293$ K, $\alpha_{10} = 0.4$, $\alpha_{1a} = 1$, $\gamma_1 = 1.4$, $\mu_1 = 1.8 \cdot 10^{-5}$ Pa · sec, $\lambda_1 = 0.025$ W/(m · sec), $R_1 = 287$ J/(kg · K) (R_1 is the gas constant), $c_v = 716$ m^2/(sec^2 · K), $r = 100$ μm, $\rho_2^0 = 2600$ kg/m^3, $c_2 = 710$ m^2/(sec^2 · K). The length and width of the duct are 1 and 0.8 m, respectively.

The boundary conditions of the problem are specified as follows: $v_i^n = 0$ and $v_i^n = v_c$ at the walls and bottom, respectively (the superscript n denotes the normal velocity components of the ith phase; $v_c(t)$ is the velocity of the bottom), and the initial conditions are specified at infinity. In the present work, two laws of motion $v_c(t)$ are considered: the duct begins to move with constant speed $v_c(t) = V_c = \text{const}$ or under the action of internal pressure p on the bottom (the Lagrange problem):

$$m\frac{dv_c}{dt} = -\int\limits_F p\,dF$$

(m and F are the mass of the duct and the bottom area, respectively). The formulated problem is solved by the method of increased stability [13]. The calculations were performed on one-dimensional and two-dimensional grids having 402 and 202×102 cells, respectively. Since numerical integration of system (1) was performed in Eulerian variables, a special algorithm, similar to the one of [14], was employed to calculate the velocity and trajectory of the movable wall (the duct bottom) and to specify the appropriate boundary conditions. The quality of the algorithm was checked by comparing the numerical and exact solutions of the classical Lagrange problem of the projection of a piston by a perfect gas. For the initial data used in the present work, the error of the computed velocity of the duct bottom was less than 0.1%.

Some Results. At the initial time, the diaphragm separating the mixture of the pressurized gas and the particles from the unperturbed gas is instantaneously removed, and this leads to decay of the discontinuity of the initial conditions. As a result, a rarefaction wave propagates from the duct exit to its bottom (to the left), and the two-phase medium flows in the opposite direction (to the right).

Within the framework of the model of a one-velocity, two-phase medium with an arbitrary concentration of the disperse phase, the solution in the field of the centered rarefaction wave is given by the formula of [9] in which the polytropic exponent should be replaced:

$$\gamma = \{\gamma_1, \gamma_2\}, \qquad \gamma_2 = (c + R)/c, \qquad c = x_1 c_v + x_2 c_2, \qquad R = x_1 R_1$$
$$(x_i = \rho_i/\rho, \qquad x_1 + x_2 = 1, \qquad i = 1, 2). \tag{2}$$

The quantity γ is equal to the adiabatic exponent of the carrier gas γ_1 in the absence of heat exchange, and it is equal to γ_2 in the limiting case of thermal equilibrium between the phases.

After removal of the diaphragm, the duct can move along the x axis under a certain law. When the duct begins to move with a constant velocity $v_c(t) = V_c = \text{const}$, the Riemann wave (2) propagates from the bottom to the exit. When the duct moves under the action of internal pressure on the bottom (the Lagrange problem), the exact solution within the framework of equilibrium representations has the form

$$x = (v + a)t + f(v), \qquad m\frac{dv_c}{dt} = -p_0\Big(1 + \frac{\gamma - 1}{2\alpha_{10}}\frac{v_c}{a_0}\Big)^{\gamma/\omega},$$
$$\frac{v_c}{a_0} = \frac{\alpha_{10}}{\omega}\Big[\Big(1 + \frac{\gamma + 1}{2\gamma}\frac{m_0}{m}\frac{a_0 t}{l}\Big)^{-(\gamma-1)/(\gamma+1)} - 1\Big], \quad \omega = \frac{\gamma - 1}{2}, \quad a_0 = \Big[\frac{\gamma p_0}{(\rho_{10} + \rho_{20})\alpha_{10}}\Big]^{1/2},$$
$$\frac{x_c}{l} = -\frac{\alpha_{10}}{\omega}\frac{a_0 t}{l} + \frac{\alpha_{10}\gamma}{\omega}\frac{m}{m_0}\Big[\Big(1 + \frac{\gamma + 1}{2\gamma}\frac{m_0}{m}\frac{a_0 t}{l}\Big)^{2/(\gamma+1)} - 1\Big] + \frac{x_0}{l}, \tag{3}$$
$$f(v) = \frac{x_0}{l} + \gamma\frac{mv}{m_0 a_0} - (1 - \alpha_{10})\frac{2\gamma}{\gamma + 1}\frac{m}{m_0}\Big[1 - \Big(1 + \frac{\omega v}{\alpha_{10} a_0}\Big)^{(\gamma+1)/(\gamma-1)}\Big].$$

Here the subscript 0 refers to the parameter values at the initial time, v is the velocity of the two-phase mixture, m_0 is the mass of the two-phase mixture in the duct at the initial time, l, v_c, x_c, and x_0 are the length of the duct (the cross-sectional area is taken to be unity), the velocity, and coordinates of the duct bottom at times $t \geqslant 0$ and at the initial time, respectively.

Usually, the validity of the above limiting schemes for calculation of wave processes in two-phase disperse media is established from estimates of the characteristic times of velocity and temperature interphase relaxations [10]:

$$t^{(v)} = (16/3)r\rho_2^0/(\rho_1^0|w_0|), \quad \mathrm{Re}_{12} > 50, \qquad t^{(\mu)} = (2/9)r^2\rho_2^0/\mu_1, \quad \mathrm{Re}_{12} < 1,$$

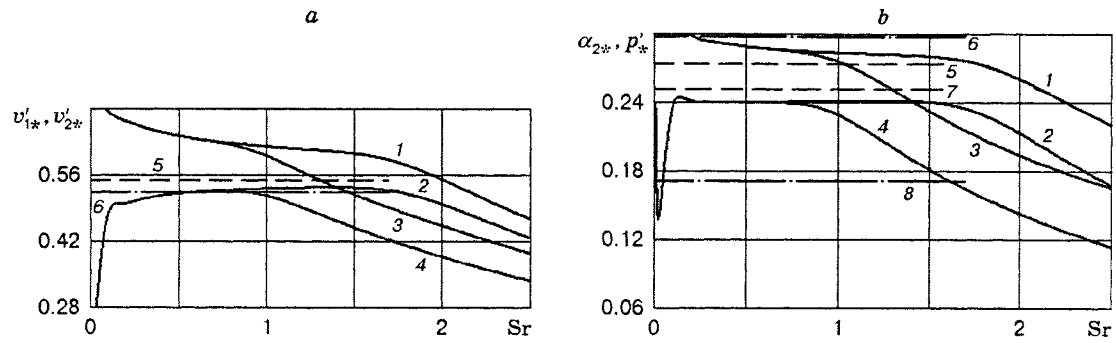

Fig. 1

$$t_i^{(T)} = r^2/æ_i^{(T)} \qquad (i = 1, 2).$$

Here $æ_i^{(T)}$ is the thermal diffusivity of the ith phase and w_0 is the characteristic magnitude of the initial slip of the phases. The temporal scale of the process considered here is the ratio of the duct length to the equilibrium velocity of sound in the two-phase mixture l/a_0. The cases where the time of the process is commensurable to or less than the characteristic time of interphase relaxation are of great interest. For example, for the initial data given above, $t^{(v)} \approx 1 \cdot 10^{-4}$ sec, $t^{(\mu)} = 3 \cdot 10^{-1}$ sec, and $l/a_0 = 1.12 \cdot 10^{-2}$ sec. The validity of equilibrium models of two-phase media for such situations requires additional investigation.

Knowledge of flow parameters at the exit from the high-pressure chamber is of practical interest. One-dimensional calculations were performed using the model of two-velocity two-temperature flow (1), and the results are compared with the above analytical solutions in the approximation of equal phase velocities.

Figure 1a shows calculated (by the nonequilibrium scheme) curves of the dimensionless velocities of the gas $v'_{1*} = v_{1*}/a_0$ and the particles $v'_{2*} = v_{2*}/a_0$ (respectively, curves 1 and 2 for an immovable duct and curves 3 and 4 for an unfastened duct moving under the action of internal pressure) versus the dimensionless time (Strouhal number) $\mathrm{Sr} = a_0 t/l$ in the cross section $[x = 0,\ v_{i*} = v_i(0, t)]$ for $m/m_0 = 1$. Here and below, the scale for the phases velocities is the initial velocity of sound a_0 in the two-phase medium in a one-velocity approximation (for $\gamma = \gamma_2$, i.e., for temperature equilibrium between the phases).

Figure 1b shows curves of the volume concentration of the disperse phase α_{2*} and the dimensionless pressure $p'_* = p_*/p_0$ (respectively, curves 1 and 2 for an immovable duct and curves 3 and 4 for an unfastened duct) at the point $x = 0$ versus the Strouhal number Sr.

Results for the limiting cases $\gamma = \gamma_1$ and $\gamma = \gamma_2$ are shown by the dashed and dot-and-dashed curves, respectively in Fig. 1. The straight lines 5 and 6 in Fig. 1a show the dimensionless critical velocity of the mixture ($v'_* = v'_{1*} = v'_{2*}$) as a function of the Strouhal number. The straight lines 5 and 6 and 7 and 8 in Fig. 1b show, respectively, the volume concentration of the particles α_{2*} and the dimensionless pressure p'_* calculated from formulas (2) as functions of the Strouhal number.

The beginning of the process is related to the characteristic time of interphase relaxation $t^{(v)}$. Next, slow equalization of the phase velocities ($l/a_0 \ll t^{(\mu)}$) continues. In this case, the flow parameters depend markedly on the Sr number, i.e., the wave pattern of the process. Thus, for an unfastened duct (Fig. 1) the duration of the quasisteady regime (analog of the critical discharge of a polytropic gas or a two-phase medium in equilibrium) is $\mathrm{Sr} \approx 1$, which is determined by arrival of the rarefaction wave related to the motion of the duct bottom.

Curves of the dimensionless velocity of the duct bottom $v'_c = v_c/a_0$ versus the Strouhal number Sr are given in Fig. 2 for $m/m_0 = 1$ and initial dimensionless pressure $P_0 = p_0/p_a = 5$ [the solid curve corresponds to a calculation by the model of a two-velocity, two-phase medium, the dashed curve shows the analytical solution (3) for the one-velocity model for $\gamma = \gamma_1$, and the dot-and-dashed curve corresponds to the equilibrium scheme for $\gamma = \gamma_2$]. With increase in the initial pressure, the difference between the calculated and analytical results decreases.

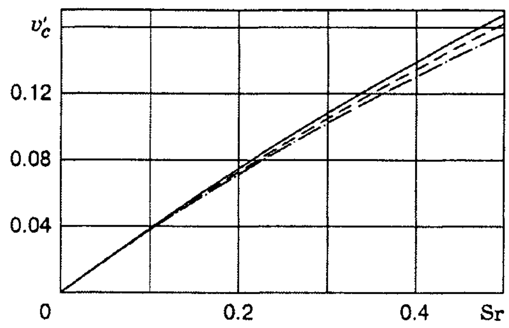

Fig. 2

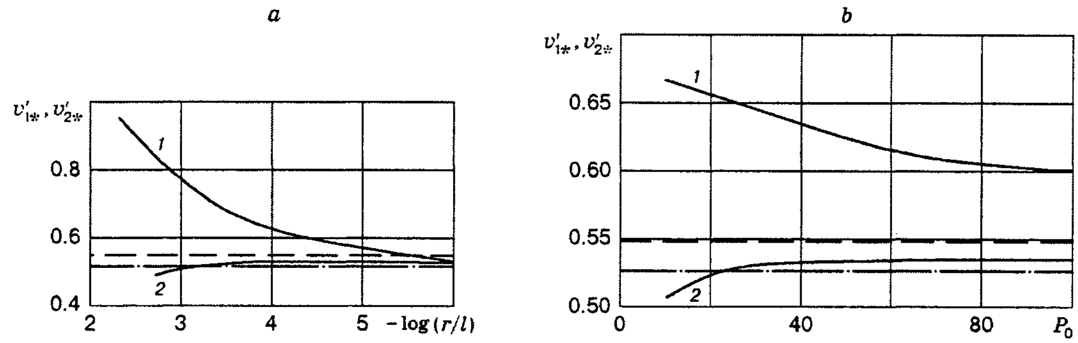

Fig. 3

The influence of the initial data of the problem, such as the radius of the dispersed particles and the initial pressure on the critical parameters of two-phase flow, in particular on the phase velocity, is also of interest. Calculations of the projection of a powder from an immovable duct are performed. Results for the critical section for Sr = 1 are given in Fig. 3 (curve 1 refers to the gas velocity, curve 2 refers the particle velocity, and the dashed and dot-and-dashed straight lines correspond to the limiting cases $\gamma = \gamma_1$ and $\gamma = \gamma_2$, respectively).

The results lead to the conclusion that over a broad range of the initial data of the problem considered (particle radius, initial pressure in the duct) and for characteristic times commensurable to or smaller than the characteristic times of velocity $t^{(\mu)}$ and thermal $t_i^{(T)}$ relaxations, models with equilibrium for velocities and temperatures apply for tentative estimation of the flow parameters.

In the case of discharge of a gas-disperse medium from a flat duct of finite dimensions, the motion of the phases is two-dimensional. In the initial stage, along with the longitudinal motion behind the duct exit, there is expansion of the two-phase medium in the transverse rarefaction waves. We determine the position in time of the transverse-wave fronts, which are characteristics of the equations of motion of the equilibrium medium:

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = a\sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}. \tag{4}$$

Here $f(x, y, t) = 0$ is the equation of the characteristics, v is the longitudinal velocity component of the equilibrium two-phase medium, and x and y are Cartesian coordinates.

We seek a self-similar solution. Besides t, x, and y, the determining parameters of the problem should include only two constants with independent dimensions [15]. In this case, these are the initial pressure p_0 and density of the mixture ρ_0. If the duct is not fixed in the longitudinal direction and begins to move under the action of internal pressure by law (3), the corresponding motion of the medium in the transverse rarefaction waves is not self-similar because of the occurrence of a third significant constant with independent dimension, i.e., the mass of the duct m.

Let us consider the case where the duct can move longitudinally with constant velocity V_c. Because $[V_c]^2 = [p_0][\rho_0]^{-1}$, the motion is self-similar.

We introduce the new independent variables $\xi = x - v_c t$, $\tau = t$, and $\zeta = y$. In view of the aforesaid, the solution of the problem depends on the two variables $z_1 = \xi/\tau$ and $z_2 = \zeta/\tau$, and (4) takes the form

$$\frac{\partial f}{\partial z_1}(v - v_c - z_1) - \frac{\partial f}{\partial z_2} z_2 = a\sqrt{\left(\frac{\partial f}{\partial z_1}\right)^2 + \left(\frac{\partial f}{\partial z_2}\right)^2}. \tag{5}$$

Using $v - v_c - a = z_1$ and $v + 2a\alpha_1/(\gamma - 1) = 2a_0\alpha_{10}/(\gamma - 1)$ and performing transformations (5), for the centered rarefaction wave [see (2)], we obtain $dz_2^2/dz_1 = -z_2^2/a + a$ or

$$\frac{dz_2^2}{dz_1} = -\frac{(\gamma + 2\alpha_1 - 1)z_2^2}{2[a_0^e - (\gamma - 1)z_1/2]} + \frac{2}{\gamma + 2\alpha_1 - 1}\left[a_0^e - \frac{(\gamma - 1)z_1}{2}\right], \quad a_0^e = a_0\left(\alpha_{10} - \frac{\gamma - 1}{2}\frac{v_c}{a_0}\right). \tag{6}$$

In contrast to the gas-dynamic problem in [16], Eq. (6) in explicit form is not integrable (since the analogy with the motion of a polytropic gas is not complete: the equation contains the volume concentration of the gas phase, which is an implicit function $\alpha_1(z_1)$ [9]).

Instead of the equation of polytrope for the two-phase medium $p(\alpha_1/\rho)^\gamma = \text{const}$, which is obtained from the equation of state for a perfect gas and the additivity of the internal energies of the phases, we use the approximating equation $p/\rho^{\gamma_a} = \text{const}$. Then, integration of (6) subject to the conditions $z_{10} = 0$ and $z_{20} = 0$ gives the well-known expression [16] in which the initial velocity of sound is replaced by the effective value a_0^e (for $\alpha_1 = \alpha_{10} = 1$), which is related to the velocity of motion of the duct:

$$z_2^2 = \frac{(\gamma_a - 1)^2}{(3 - \gamma_a)(\gamma_a + 1)}\left(\frac{2}{\gamma_a - 1}a_0^e - z_1\right)^{(\gamma_a+1)/(\gamma_a-1)}$$

$$\times\left[\left(\frac{2}{\gamma_a - 1}a_0^e - z_1\right)^{-(3-\gamma_a)/(\gamma_a-1)} - \left(\frac{2}{\gamma_a - 1}a_0^e\right)^{-(3-\gamma_a)/(\gamma_a-1)}\right]. \tag{7}$$

Equation (7) describes the front of the transverse rarefaction wave in the range $0 \leqslant z_1 \leqslant z_1' = v' - v_c - a'$, where the prime denotes the parameter values on the limiting characteristic of the centered rarefaction wave [see (2)].

The fronts of the transverse rarefaction waves in the region of constant two-phase flow $v' - v_c - a' < z_1 \leqslant z_1'' = v'' - v_c$ and in the postshock concurrent flow $v'' - v_c < z_1 \leqslant z_1''' = D - v_c$ are given, respectively, by

$$z_2^2 = (z_2' - a'^2)e^{-(z_1 - z_1')/a'} + a'^2, \quad z_2^2 = (z_2'' - a_1''^2)e^{-(z_1 - z_1'')/a_1''} + a_1''^2, \tag{8}$$

where D and a_1 are the velocities of the shock wave and sound in the gas; the two primes correspond to the values on the contact surface.

To estimate the validity of the approximate solution, we calculated the relative error (in percent) of the Mach number ($M_1 = D/a_1$) of the shock wave formed as a result of decay of the discontinuity in the system "disperse mixture–gas" for $\gamma_a = 2.35$. The results are given in Table 1.

The initial two-dimensional stage of discharge of the two-phase medium is calculated by the nonequilibrium model (1) and the results are compared with the analytical results obtained above within the framework of a one-velocity approximation. Figure 4 shows a fragment of the velocity field of the disperse phase (for volume concentrations higher than 3% of the initial value) at $t = 0.0015$ sec. The vertical lines show the characteristic surfaces (obtained analytically): ξ^0 is the duct exit, ξ' is the beginning of the region of constant flow of the two-phase medium, ξ'' is the interface between the media, and the postshock concurrent gas flow is located to the right of the interface. The solid curve shows the fronts of the transverse rarefaction waves calculated from formulas (7) and (8). The calculations were performed for the following initial data: $p_0 = 7.5$ MPa and $V_c = -25$ m/sec; the remaining values are given above in the formulation of the problem.

The ordinate of the transverse rarefaction wave front (7) adjacent to the one-dimensional centered Riemann wave can have a minimum, depending on the initial pressure ratio p_0/p_a and the duct velocity V_c.

TABLE 1

p_0/p_a	ΔM_1, %
10	0.22
20	0.11
30	0.04
40	0.20
50	0.36
60	0.51
70	0.66
80	0.81
90	0.95
100	1.08

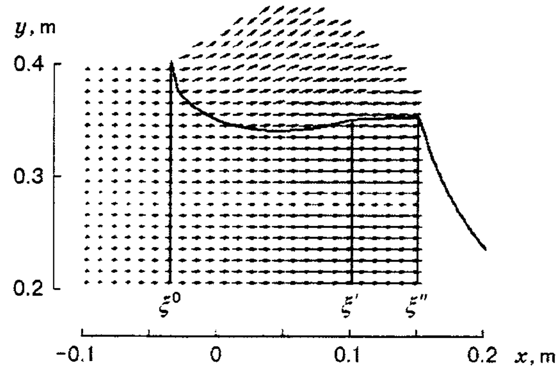

Fig. 4

TABLE 2

p_0/p_a	$V_c = 0$		$V_c = -10$ m/sec		$V_c = -25$ m/sec	
	z_1^*	z_1'	z_1^*	z_1'	z_1^*	z_1'
10	19.60	5.946	23.22	15.95	28.63	30.95
15	24.01	13.23	27.62	23.23	33.04	38.23
20	27.72	19.63	31.34	29.63	36.75	44.63
25	31.00	25.43	34.61	35.43	40.02	50.43
30	33.96	30.77	37.57	40.77	42.99	55.77
35	36.68	35.75	40.29	45.75	45.70	60.75
40	39.21	40.43	42.82	50.43	48.24	65.43

Table 2 gives values of z_1^* (the value of z_1 for which z_2 has a minimum) and z_1' (the position of the limiting characteristic) for various values of p_0/p_a and V_c. Obviously, the ordinate of the transverse rarefaction wave front has a minimum with the proviso that $z_1^* < z_1'$.

Thus, the structure of the initial stage of unsteady discharge from a movable, bounded, flat duct can be described as follows: in the central zone, the flow of the gas and the two-phase medium is one-dimensional over a finite period of time, and transverse rarefaction waves are located from above and from below. In this case, if $z_1' > 0$, all the three characteristic transverse rarefaction waves are present: in the region of the centered rarefaction wave in the two-phase medium (7), in the zone of constant flow of the two-phase medium, and in the postshock concurrent gas flow (8). When $z_1' \leqslant 0$ and $z_1'' > 0$, two-dimensional flow is formed only by two waves, whose fronts are described by (8). This is the case, for example, with a subcritical initial difference in pressure and in an immovable duct. Formally, it is also possible to indicate different discharge patterns, for example, when a lateral rarefaction wave is formed only in the "pure" gas.

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